



PATHWAY 2 GEOMETRY
Education with Destination

Pathway2 Geometry





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			Cluster	Occupation
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			Cluster	Occupation
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LESSON 1.3

Use Theorems About Angles



CAREER SPOTLIGHT: Carpenter

Occupation Description

Carpenters construct, repair, and install building frameworks and structures made from wood and other materials.

Carpenters have many different tasks. Some carpenters insulate office buildings; others install drywall or kitchen cabinets in homes. Still others focus on production or commercial work to help construct tall buildings or bridges. These carpenters also erect shoring and scaffolding for buildings.

Education

Carpenters typically need a high school diploma and learn on the job or through apprenticeships. Certain high school courses, such as mathematics and mechanical drawing, may be useful. Some vocational-technical schools offer associate's degrees in carpentry. The programs vary in length and teach basics and specialties in carpentry.

Potential Employers

The largest employers of carpenters are as follows:

Self-employed workers	27%
Residential building construction	22%
Nonresidential building construction	13%
Building finishing contractors	12%
Foundation, structure, and building exterior contractors	10%

Watch a video about carpenters:

<https://cdn.careeronestop.org/OccVids/OccupationVideos/47-2031.00.mp4>

Career Cluster

Architecture & Construction

Career Pathway

Construction

Career Outlook

- Salary Projections:
Low-End Salary, \$30,170
Median Salary, \$48,330
High-End Salary, \$84,690
- Jobs in 2018: 1,006,500
- Job Projections for 2028:
1,086,600 (increase of 8%)

Geometry Concept

- Apply theorems about angles.

Is this a good career for me?

Carpenters:

- Follow blueprints and building plans to meet the needs of clients
- Measure, cut, and shape wood, plastic, and other materials
- Construct and install building frameworks, including walls, floors, and doorframes
- Instruct and direct laborers and other construction helpers
- Install structures and fixtures, such as windows and molding
- Inspect and replace damaged framework or other structures and fixtures

Lesson Objective

In this lesson, you will look at how a carpenter applies theorems about angles to carpentry tasks such as framing buildings and building furniture.

Definitions

Angles that have the same angle measure are **congruent angles**.

A **linear pair** are two adjacent angles formed by intersecting lines.

Vertical angles are opposite angles formed by intersecting lines.

A pair of angles are **complementary** if the sum of the angle measures is 90° .

A pair of angles are **supplementary** if the sum of the angle measures is 180° .

Theorems

Linear Pair Theorem: Two angles that form a linear pair are supplementary.

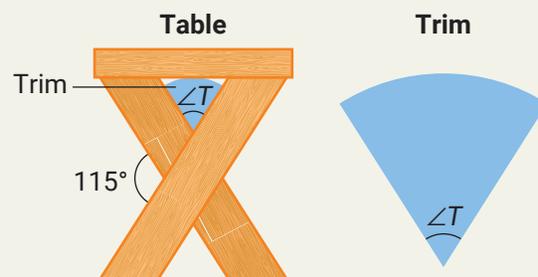
Vertical Angles Theorem: Vertical angles are congruent.

Congruent Complements Theorem: If two angles are complements of the same angle (or congruent angles), then the two angles are congruent.

Congruent Supplements Theorem: If two angles are supplements of the same angle (or congruent angles), then the two angles are congruent.

1 Step Into the Career: Linear Pair Theorem

A carpenter is building coffee tables with crossed legs and decorative trim. One version is shown. What is the angle measure that the carpenter should cut the bottom angle of the trim, $\angle T$, so that it fits with no gaps?



Devise a Plan

Step 1: Model the table legs with a pair of intersecting lines.

Step 2: Identify the type of angles formed by the 115° angle and $\angle T$.

Step 3: Write an equation to describe the connection between the 115° angle and $\angle T$.

Step 4: Solve for $m\angle T$.

Walk Through the Solution

Step 1: Model the table legs with a pair of intersecting lines, as shown.

Step 2: The 115° angle and $\angle T$ are adjacent angles formed by intersecting lines. They are a linear pair.

Step 3: By the Linear Pair Theorem, the angles are supplementary. The sum of the measures of the 115° angle and $\angle T$ is 180° .

$$115^\circ + m\angle T = 180^\circ$$

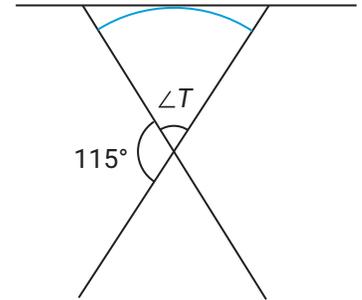
Step 4: Solve for $m\angle T$.

$$115^\circ + m\angle T = 180^\circ$$

$$m\angle T = 180^\circ - 115^\circ$$

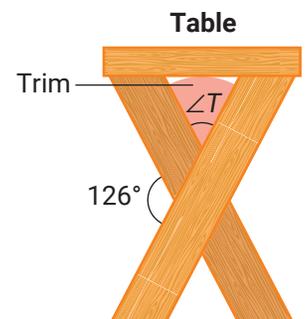
$$m\angle T = 65^\circ$$

The carpenter should cut the trim piece so that $m\angle T$ is 65° .



On the Job: Apply The Linear Pair Theorem

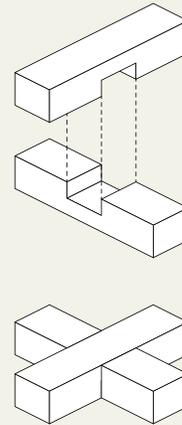
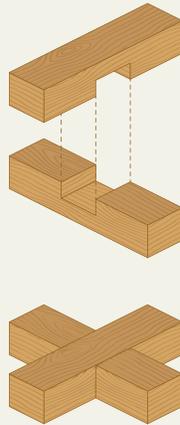
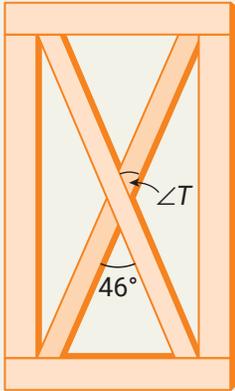
1. A carpenter building coffee tables has decorative trim cut to various angles.
 - a. Will decorative trim cut with an angle of 57° fit the table shown?
 - b. If the trim cut to an angle of 57° does not fit the table shown, what angle measure should the carpenter cut the angle of the trim, $\angle T$, so that it fits with no gaps?



2 Step Into the Career: Vertical Angles Theorem

A carpenter is building a shelving unit with additional support on each side provided by two boards that are connected with edge cross lap joints, as shown.

What is the measure of $\angle T$ that the carpenter should cut the notch in the board?



In edge cross lap joints, matching notches are cut in the two boards so that they fit together snugly.

Devise a Plan

Step 1: Model the two boards with a pair of intersecting lines.

Step 2: Identify the type of angles formed by the 46° angle and $\angle T$.

Step 3: Identify the relationship between the measure of the 46° angle and $\angle T$.

Step 4: Find $\angle T$.

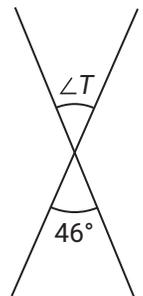
Walk Through the Solution

Step 1: Model the two boards with a pair of intersecting lines as shown.

Step 2: The 46° angle and $\angle T$ are opposite angles formed by intersecting lines. They are vertical angles.

Step 3: By the Vertical Angles Theorem, vertical angles formed by intersecting lines are equal.

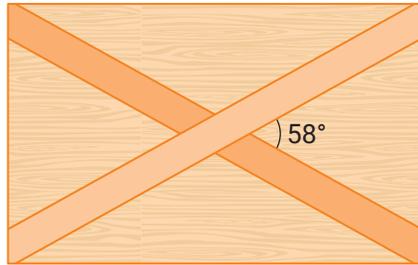
Step 4: Since $\angle T$ and the 46° angle are vertical angles, $m\angle T = 46^\circ$ by the Vertical Angles Theorem.



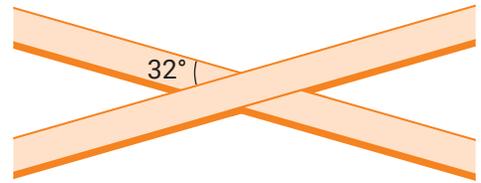
The carpenter should cut the notch with a 46° angle.

On the Job: Apply The Vertical Angles Theorem

2. A carpenter's apprentice has cut notches into several boards for edge cross lap joints to go on the sides of the box shown.

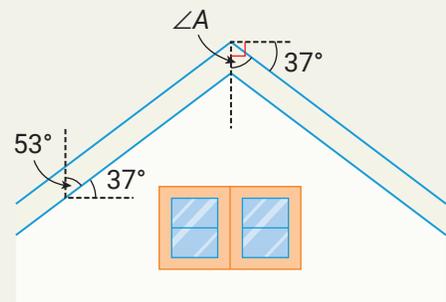


- If the apprentice cut the boards using the angle shown at the right, do the boards work for the box?
- If the boards that the apprentice cut do not fit on sides of the box, what angle should have been used for the notches?
- Can you identify the error that the apprentice made when calculating the angle that should be used?



3 Step Into the Career: Congruent Complements Theorem

A carpenter is framing the roof of a house where the pitch of the roof is 37° , as shown. What angle measure should be used for $\angle A$?



Devise a Plan

There are two 37° angles, a 53° angle, and $\angle A$ involved.

Step 1: Identify the relationship between the 37° angle and the 53° angle.

Step 2: Identify the relationship between the 37° angle and $\angle A$.

Step 3: Identify the relationship between the 53° angle and $\angle A$.

Step 4: Find $m\angle A$.

Walk Through the Solution

Step 1: Since $37 + 53 = 90$, the 37° angle and the 53° angle form a right angle. They are complementary angles.

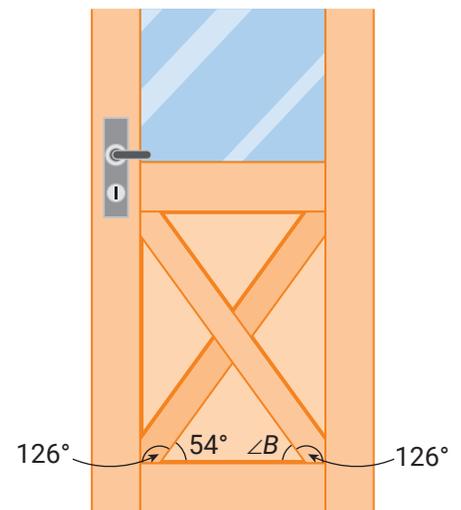
Step 2: The diagram shows that $\angle A$ and the 37° angle form a right angle, so they are complementary angles.

Step 3: From Steps 1 and 2, $\angle A$ and the 53° angle are both complements of 37° angles. By the Congruent Complements Theorem, two angles that are complements of the same angle (or congruent angles) are congruent.

Step 4: Since $\angle A$ is congruent to the 53° angle, $m\angle A$ is 53° .

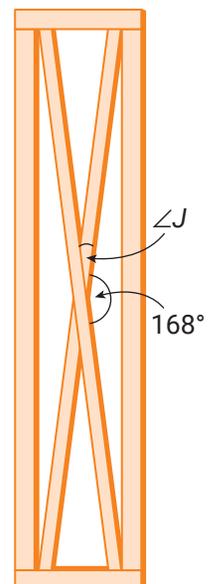
On the Job: Apply The Congruent Supplements Theorem

3. A carpenter is repairing a wooden door and needs to replace a triangular panel. What angle measure should be used for $\angle B$?
- What is the relationship between a 54° angle and a 126° angle?
 - What is the relationship between $\angle B$ and the adjacent 126° angle?
 - What is $m\angle B$?



Career Spotlight: Practice

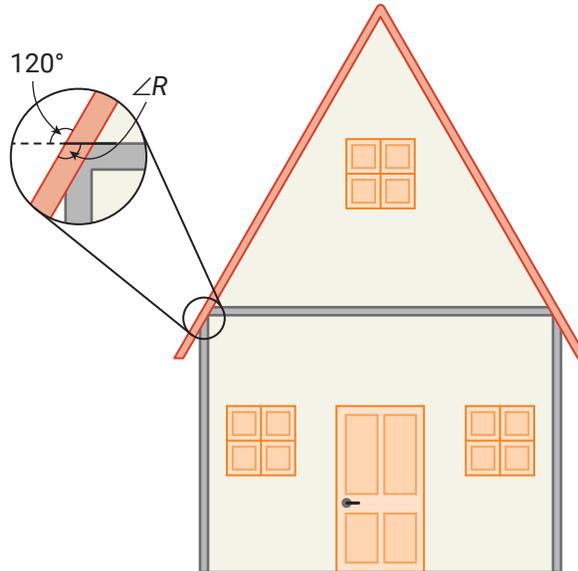
4. A carpenter is using an edge cross lap joint to create an X-shaped brace for the side of a bookcase. What angle measure should the carpenter use for $\angle J$ so that the boards fit together with no gaps?



5. A carpenter needs rafter ties to frame a very steep roof. The outside line of the roof makes a 120° angle with a horizontal line. What is the measure of $\angle R$?

QUICK TIP

Rafter ties are horizontal pieces that connect two sides of a roof.



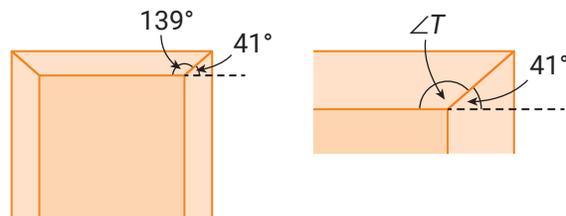
Devise a Plan

Step 1: Determine the relationship between the 120° angle and $\angle R$.

Step 2: _____ ? _____.

Step 3: _____ ? _____.

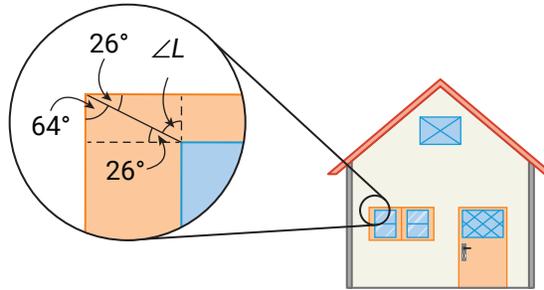
6. A carpenter needs to replace the trim above a doorway where the top trim and the side trim are different widths. The angles measured are shown.



- What is the relationship between the 139° angle and the 41° angle shown on the left?
- What is the relationship between the 41° angle and $\angle T$ shown on the right?
- What relationship can you find between the 139° angle and $\angle T$?
- What is $m\angle T$?

Career Spotlight: Check

7. A carpenter is working on a house. For the shutters on the lower window, the side frame needs to be wider than the top frame, so that the shutters can be supported by the side frame.



Select the answer from each box that makes the sentence true.

The 26° angle and the 64° angle are

- a. supplementary angles
b. complementary angles
c. a linear pair

. $\angle L$ and the

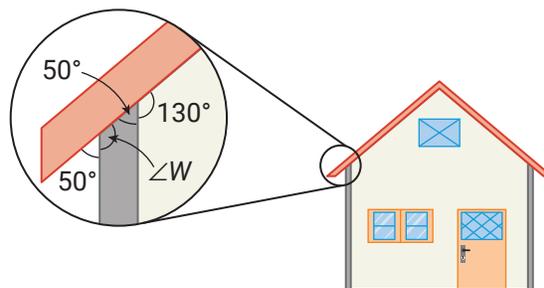
26° angle are

- a. supplementary angles
b. complementary angles
c. a linear pair

. The measure of $\angle L$ is

- a. 26°
b. 64°
c. 116°

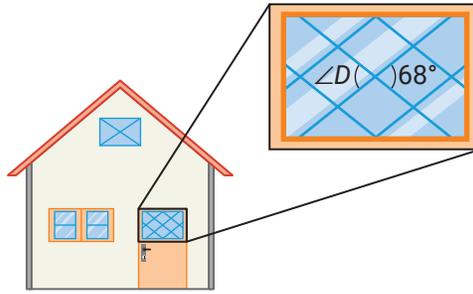
8. A carpenter is working on a house. The roof and the outer walls meet to form several angles.



What is $m\angle W$, and what theorem could you use to determine this?

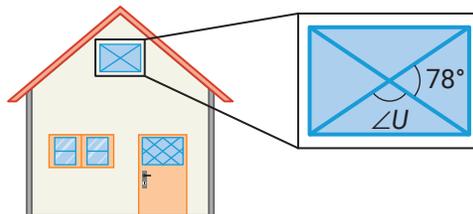
- A. $m\angle W = 50^\circ$, Congruent Supplements Theorem
B. $m\angle W = 130^\circ$, Congruent Complements Theorem
C. $m\angle W = 50^\circ$, Congruent Complements Theorem
D. $m\angle W = 130^\circ$, Congruent Supplements Theorem

9. A carpenter is working on a house. The window in the door features framing that creates many angles.



Select all the statements that are true.

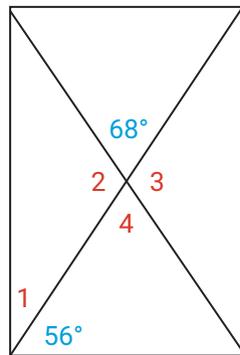
- a. $m\angle D = 112^\circ$
 - b. $\angle D$ and the angle labeled 68° are supplementary angles.
 - c. $m\angle D = 68^\circ$
 - d. $\angle D$ and the angle labeled 68° are complementary angles.
 - e. $\angle D$ and the angle labeled 68° are vertical angles.
 - f. $m\angle D = 22^\circ$
 - g. $\angle D$ and the angle labeled 68° are a linear pair of angles.
10. A carpenter is working on a house. The upper window has a frame that creates angles.



Select all the statements that are true.

- a. $m\angle U = 102^\circ$
- b. $\angle U$ and the angle labeled 78° are supplementary angles.
- c. $m\angle U = 78^\circ$
- d. $\angle U$ and the angle labeled 78° are complementary angles.
- e. $\angle U$ and the angle labeled 78° are vertical angles.
- f. $m\angle U = 12^\circ$
- g. $\angle U$ and the angle labeled 78° are a linear pair of angles.

11. A carpenter is building a door with an X panel like the one shown. The panel is modeled by the diagram. The panel is a rectangle, so the corners are right angles. Use the diagram to find the measures of the numbered angles.



Match each angle with the correct angle measure.

	34°	56°	68°	112°
∠1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
∠2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
∠3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
∠4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

LESSON 3.2

Apply Reflections



CAREER SPOTLIGHT: Marine Engineer and Naval Architect

Occupation Description

Marine engineers and naval architects design, build, and maintain ships, from aircraft carriers to submarines and from sailboats to tankers. Marine engineers are also known as marine design engineers or marine mechanical engineers and are responsible for the internal systems of a ship, such as the propulsion, electrical, refrigeration, and steering systems. Naval architects are responsible for the ship design, including the form, structure, and stability of hulls.

Education

Programs in marine engineering and naval architecture typically include courses in calculus, physics, computer-aided design, fluid mechanics, ship hull strength, and mechanics of materials. Most programs also include time at sea, where students gain hands-on experience on a vessel.

Some marine engineering and naval architecture programs are offered at state maritime academies.

Potential Employers

The largest employers of marine engineers and naval architects are as follows:

Engineering services	25%
Federal government, excluding postal service	9%
Other professional, scientific, and technical services	7%
Transportation and warehousing	4%

Watch a video about marine engineers and naval architects: <https://cdn.careeronestop.org/OccVids/OccupationVideos/17-2121.00.mp4>

Career Cluster

Science, Technology, Engineering & Mathematics

Career Pathway

Engineering and Technology

Career Outlook

- Salary Projections:
Low-End Salary, \$65,440
Median Salary, \$92,400
High-End Salary, \$147,710
- Jobs in 2018: 11,700
- Job Projections for 2028: 12,700 (increase of 9%)

Geometry Concepts

- Describe reflections on a coordinate plane using functions.
- Apply reflections on a coordinate plane

Is this a good career for me?

Marine engineers and naval architects:

- Prepare system layouts and detailed drawings
- Inspect marine equipment and machinery
- Design and oversee the testing, installation, and repair of marine equipment
- Prepare cost estimates, contract specifications, and schedules

Lesson Objective

In this lesson, you will look at how a marine engineer or naval architect applies geometric reflections on a coordinate plane.

Reflections on the Coordinate Plane

A **reflection** is a transformation that results in a mirror image of every point in a figure across a line known as the **line of reflection**.

The function $R_{x\text{-axis}}$ reflects a point across the x -axis.

$$R_{x\text{-axis}}(x, y) = (x, -y)$$

The function $R_{y\text{-axis}}$ reflects a point across the y -axis.

$$R_{y\text{-axis}}(x, y) = (-x, y)$$

The function $R_{y=x}$ reflects a point across the line $y = x$.

$$R_{y=x}(x, y) = (y, x)$$

The function $R_{y=-x}$ reflects a point across the line $y = -x$.

$$R_{y=-x}(x, y) = (-y, -x)$$

The composite function $R_{y\text{-axis}} \circ R_{x\text{-axis}}$ reflects a point across the x -axis and then across the y -axis.

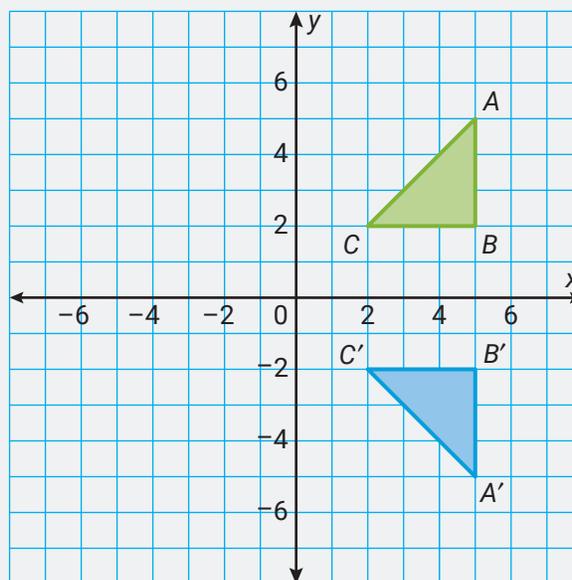
$$(R_{y\text{-axis}} \circ R_{x\text{-axis}})(x, y) = (-x, -y)$$

The coordinate plane shows $\triangle ABC$ and its reflection across the x -axis, $\triangle A'B'C'$.

$$R_{x\text{-axis}}(A) = R_{x\text{-axis}}(5, 5) = (5, -5) = A'$$

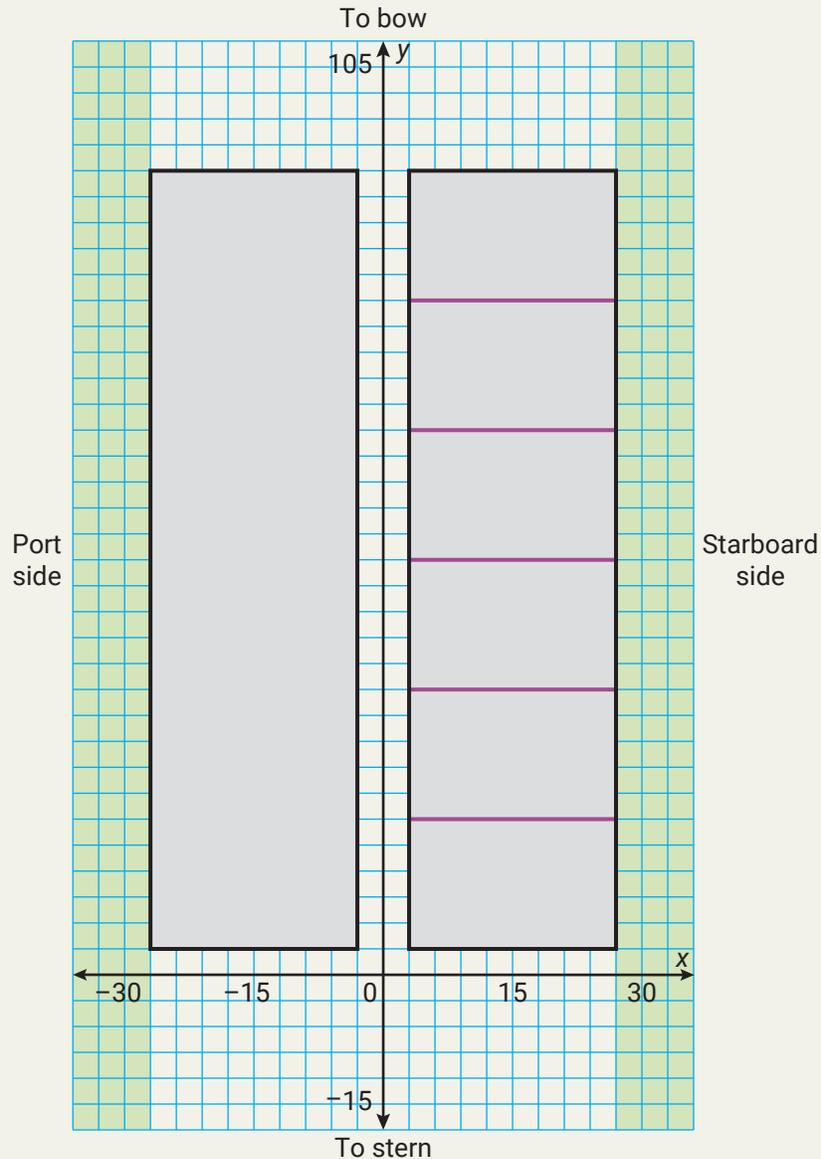
$$R_{x\text{-axis}}(B) = R_{x\text{-axis}}(5, 2) = (5, -2) = B'$$

$$R_{x\text{-axis}}(C) = R_{x\text{-axis}}(2, 2) = (2, -2) = C'$$



1 Step Into the Career: Reflection Functions

Danica is a naval architect. She is sketching a plan for the hinges of the hatch covers of a cargo ship when the hatch covers are in the closed position. The hinges on the starboard side of the ship are shown. The hinges on the port side of the ship are a reflection across the y -axis. Draw the hatch cover hinges for the port side of the ship when the hatch covers are in the closed position.



Devise a Plan

- Step 1:** Determine which coordinate is affected by the reflection across the y -axis.
- Step 2:** Determine the coordinates of the endpoints of the port hatch cover hinge closest to the stern of the ship.
- Step 3:** Draw the hatch cover hinges for the port side of the ship.

Walk Through the Solution

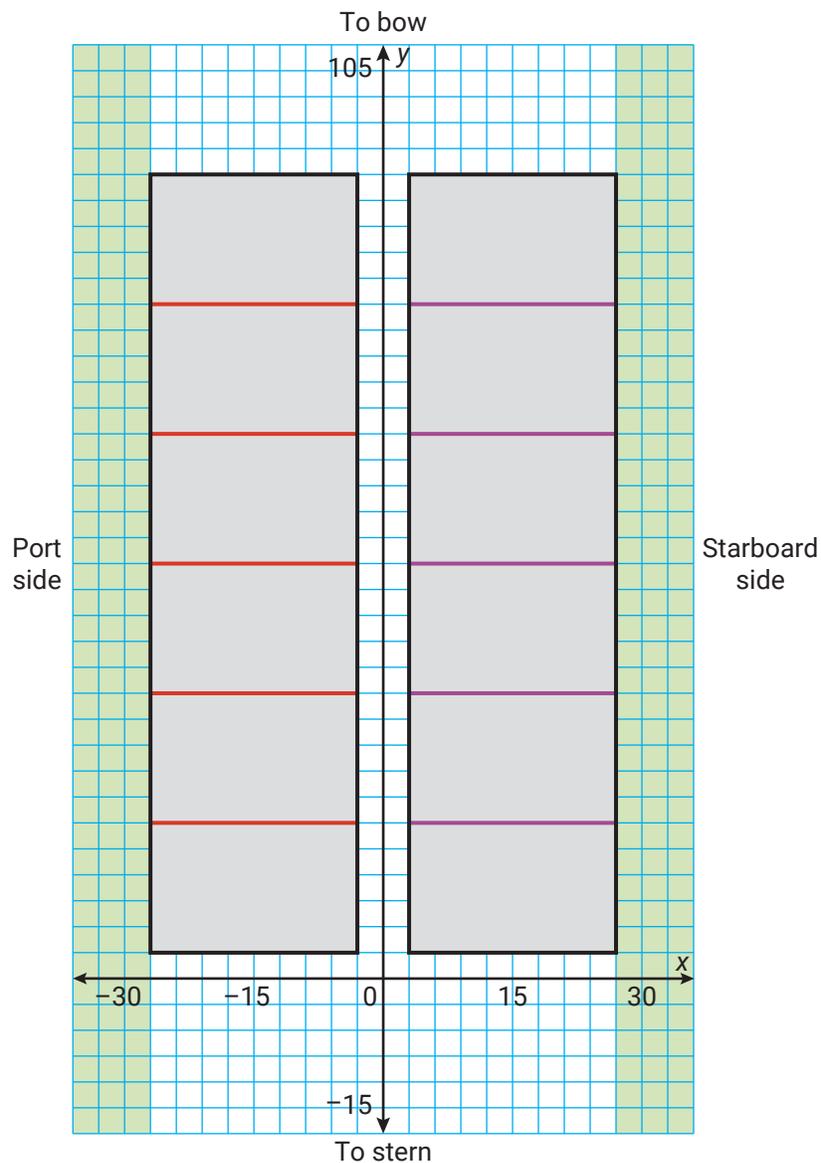
Step 1: Determine which coordinate is affected by the reflection across the y -axis.

A reflection across the y -axis affects the x -coordinate.

Step 2: Determine the coordinates of the endpoints of the port hatch cover hinge closest to the stern of the ship.

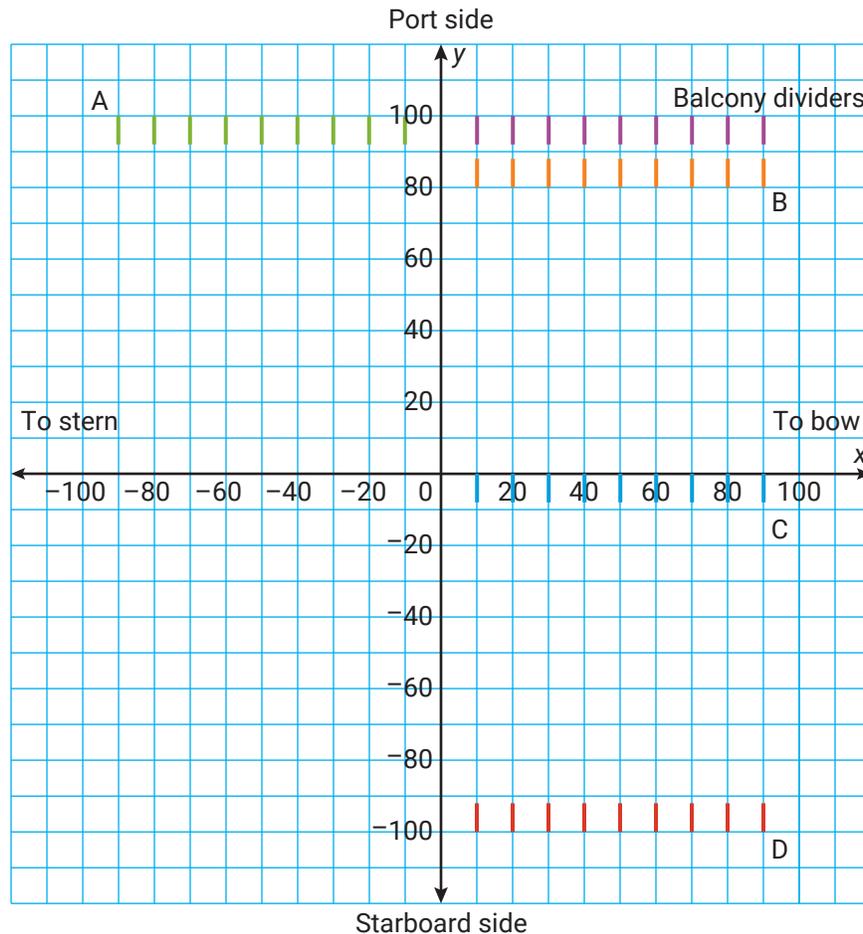
The coordinates of the endpoints of the starboard hatch cover hinge closest to the stern of the ship are $(3, 18)$ and $(27, 18)$. The transformation is $R_{y\text{-axis}}(x, y) = (-x, y)$, so the coordinates of the endpoints of the port hatch cover hinge closest to the stern of the ship are $(-3, 18)$ and $(-27, 18)$.

Step 3: Draw the hatch cover hinges for the port side of the ship.



On the Job: Apply Reflection Functions

- The plans for a cruise ship are shown on a coordinate plane. A naval architect is reviewing the position of balcony dividers for the cruise ship. The dividers on the port side of the ship are shown labeled as A. The dividers on the starboard side of the ship are a reflection across the x-axis. Which reflection shows the balcony dividers for the starboard side of the ship?



2 Step Into the Career: Reflections on the Coordinate Plane

A marine engineer is working on plans to attach chainplates to a sailboat. His plans are placed on a coordinate plane. The coordinates of the starboard chainplates are $(1, -5)$, $(2, -4)$, and $(3, -3)$. The coordinates of the port chainplates can be found using the reflection $R(x, y) = (y, x)$. What are the coordinates of the port chainplates? What is the line of reflection?



Devise a Plan

Recall that in a reflection, the line of reflection is the perpendicular bisector of the line segment between a point and its image.

Step 1: Determine the coordinates of the port chainplates.

Step 2: Plot both sets of coordinates, connect each point to its reflection, and then construct the perpendicular bisector of each segment.

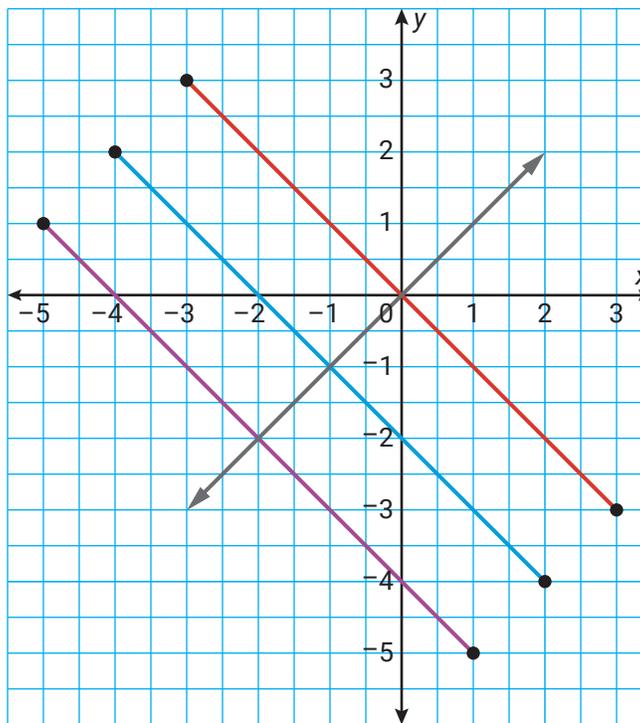
Step 3: Identify the line of reflection.

Walk Through the Solution

Step 1: Determine the coordinates of the port chainplates.

The reflection swaps the position of the x - and y -coordinates, so the coordinates of the port chainplates are $(-5, 1)$, $(-4, 2)$, and $(-3, 3)$.

Step 2: Plot both sets of coordinates, connect each point to its reflection, and then construct the perpendicular bisector of each segment.



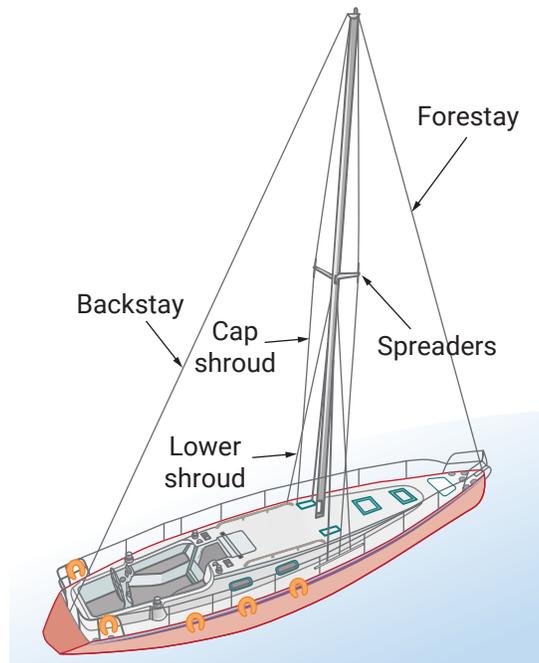
Step 3: Identify the line of reflection.

From the graph, you can see all of the perpendicular bisectors lie on the line $y = x$. Because the perpendicular bisectors are the same line, the line $y = x$ is the line of reflection.

The coordinates of the port chainplates are $(-5, 1)$, $(-4, 2)$, and $(-3, 3)$. The line of reflection is $y = x$.

On the Job: Apply Reflections on the Coordinate Plane

2. Zoey is a marine engineer. She is working on plans to attach a spreader to a mast. Her plans are on a coordinate plane. The cap shroud, one of the rigging cables, extends from the top of the mast to each end of the spreader and down to the chainplate on each side of the boat. The end of the spreader on the starboard side is at $(80, 600)$, and the starboard chainplate is at $(110, 0)$. The mast is drawn across the y -axis. If the mast is the line of reflection, what are the coordinates of the end of the spreader on the port side and of the port chainplate?

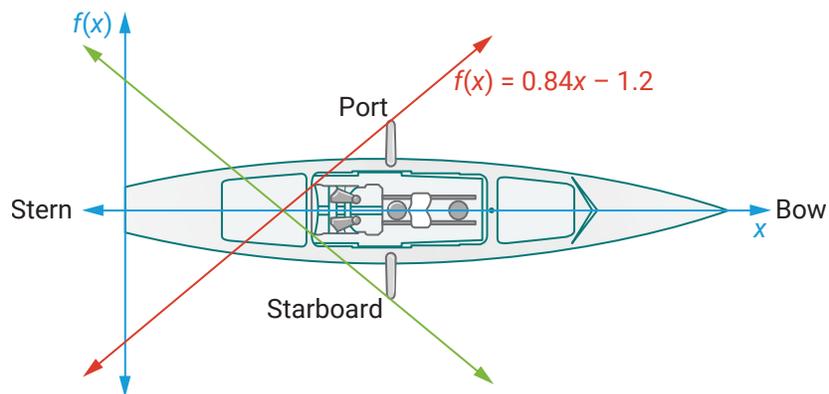


Career Spotlight: Practice

3. A marine engineer is planning to construct a sculling shell, which is a type of row boat, as shown in the diagram. In the diagram, the port oar bracket will follow the line with equation $y = 0.84x - 1.2$. The starboard oar bracket will follow the line reflected across the x -axis. What is an equation of the line that the starboard oar bracket follows?

QUICK TIP

Identify two points on the given line, and reflect them across the x -axis. Then write an equation of the line using the two reflected points.



4. A naval architect is sketching a portion of the deck of an aircraft carrier on a coordinate plane. There is enough room for the jets to park diagonally on both sides of a taxiway. On one side, corners of the surface are located at $(125, 0)$, $(150, 250)$, and $(200, 300)$. Other corners can be found using the reflection $R(x, y) = (150 - x, y)$.



- a. What are the coordinates of the other corners?
b. What is the line of reflection?

Devise a Plan 

Step 1: Determine the coordinates of the other corners.

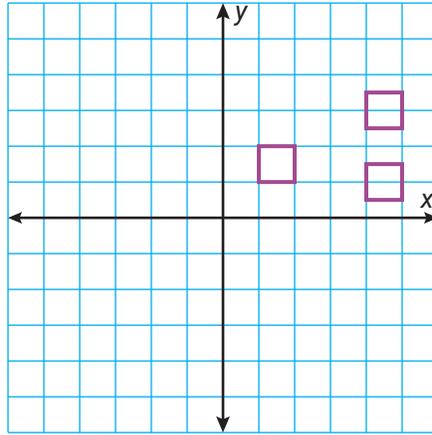
Step 2: _____ ?

Step 3: _____ ?

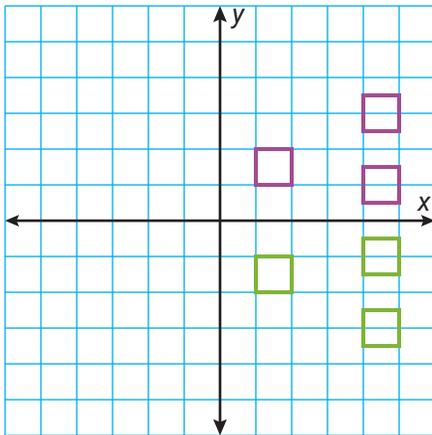
5. A marine engineer has placed the overhead view of a ship on a coordinate plane. The ship is symmetric across the line $y = -x$. The plans show one part of the port side of the ship from the point $(-45, 30)$ to the point $(-30, 20)$. What are the endpoints of the part of the starboard side that is the reflection across $y = -x$?

Career Spotlight: Check

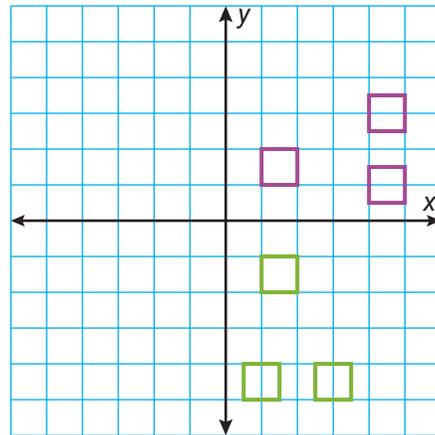
6. Antonio is a naval architect. He is reviewing a design of a dining room for a large ship. The designer has noted that the dining tables shown are reflections across the line $y = x$ followed by reflections across the y -axis. Which layout shows the architect's intentions?



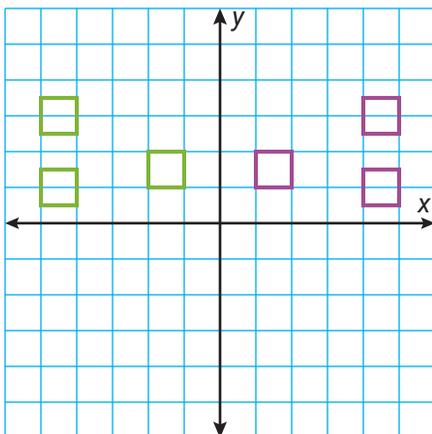
A.



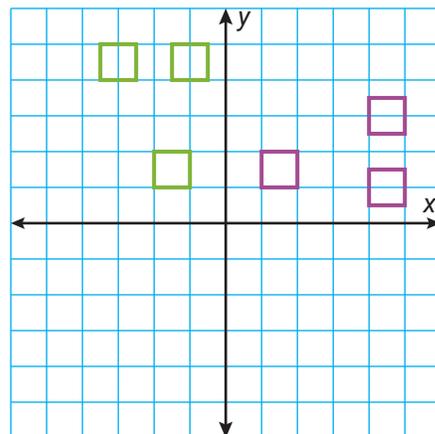
C.



B.



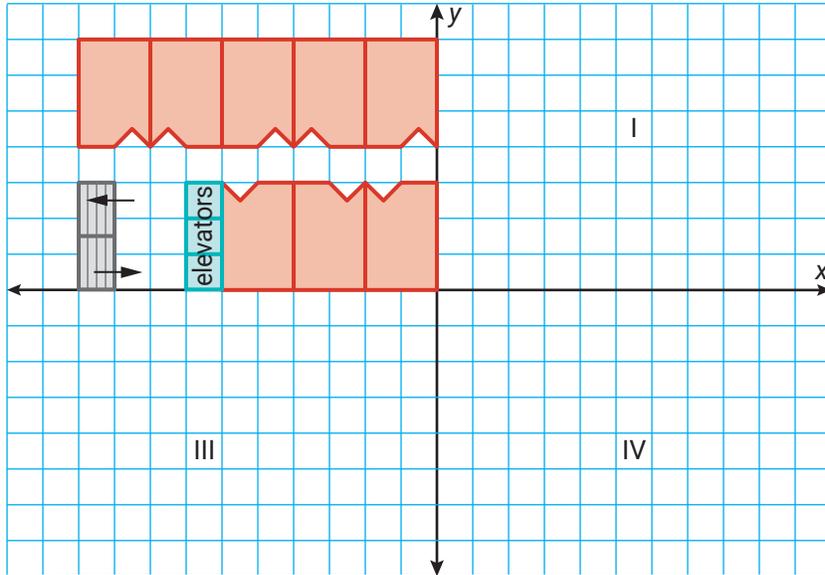
D.



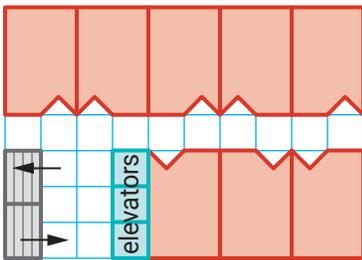
7. A naval architect is reviewing a designer's deck plan to learn where the elevators are going to be on a cruise ship. The designer intends to show the elevators in quadrant II of the plans reflected across the x-axis, across the y-axis, and across both the x- and y-axes. Where does each of the pieces shown belong?

Fill in the blank by selecting the correct location from the panel.

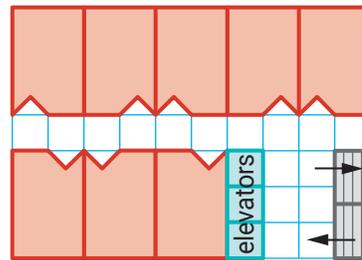
quadrant I	quadrant IV
quadrant III	no quadrant



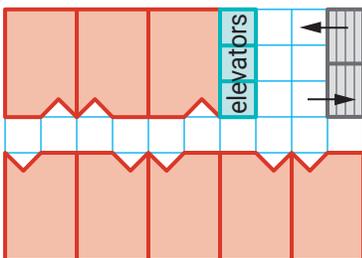
Piece 1: _____



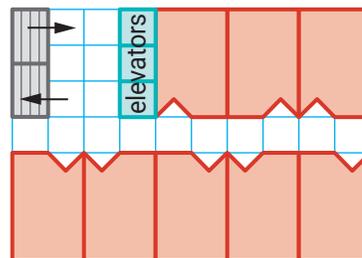
Piece 3: _____



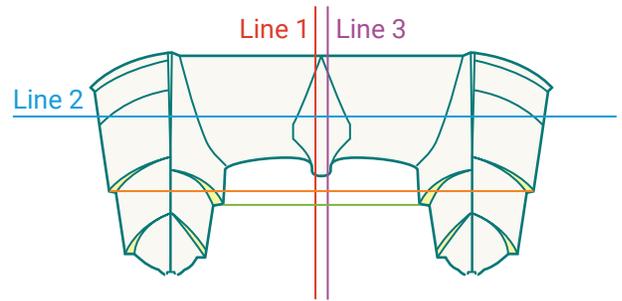
Piece 2: _____



Piece 4: _____

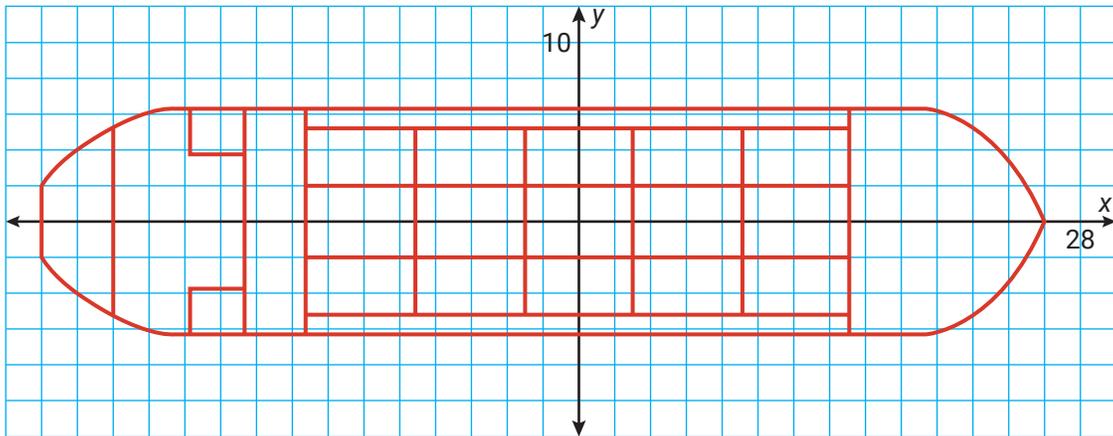


8. A marine engineer has designed this catamaran hull. When in the water, it will be perfectly balanced. Identify the line of reflection.



- A. Line 1 is the line of reflection.
- B. Line 2 is the line of reflection.
- C. Line 3 is the line of reflection.
- D. There is no line of reflection.

9. Regulations require separated ballast tanks in various locations on a ship. Morgan is a naval architect reviewing the design plans of an oil tanker to learn where the separated ballast tanks will be. One location has corners $(-9, -2)$, $(-3, -2)$, $(-9, -5.5)$, and $(-3, -5.5)$. Other locations can be found using the reflections $R_1(x, y) = (-x, y)$, $R_2(x, y) = (x, -y)$, and $(R_1 \circ R_2)(x, y) = (-x, -y)$. What are the coordinates of the corners of the other locations? What are the lines of reflection?



Select all the statements that are true.

- a. One location has corners $(9, 2)$, $(3, 2)$, $(9, 5.5)$, and $(3, 5.5)$.
- b. One location has corners $(9, -2)$, $(3, -2)$, $(9, -5.5)$, and $(3, -5.5)$.
- c. One location has corners $(9, -2)$, $(-3, 2)$, $(9, 5.5)$, and $(-3, -5.5)$.
- d. One location has corners $(-9, 2)$, $(-3, 2)$, $(-9, 5.5)$, and $(-3, 5.5)$.
- e. One line of reflection is the x -axis.
- f. One line of reflection is the y -axis.

LESSON 11.5

Volume of Cylinders, Cones, and Spheres



CAREER SPOTLIGHT: Agricultural Engineer

Occupation Description

Agricultural engineers attempt to solve agricultural problems concerning power supplies, the efficiency of machinery, the use of structures and facilities, pollution and environmental issues, and the storage and processing of agricultural products.

Agricultural engineers work in farming, including aquaculture (farming of seafood), forestry, and food processing. They work on a wide variety of projects. For example, some agricultural engineers work to increase the storage capacity and efficiency of refrigeration. Those with computer programming skills work to integrate artificial intelligence and geospatial systems into agriculture.

Education

Students interested in agricultural engineering take courses in advanced calculus, physics, biology, and chemistry. They also may take courses in business, public policy, and economics.

Potential Employers

The largest employers of agricultural engineers are as follows:

Federal government, excluding postal service	20%
Colleges, universities, and professional schools; state	12%
Management, scientific, and technical consulting services	9%
Engineering services	6%

Watch a video about agricultural engineers:

<https://cdn.careeronestop.org/OccVids/OccupationVideos/17-2021.00.mp4>

Career Cluster

Agriculture, Food & Natural Resources

Career Pathway

Power, Structure & Technical Systems

Career Outlook

Salary Projections:

- Low-End Salary, \$47,330
Median Salary, \$80,720
High-End Salary, \$160,950
- Jobs in 2018: 2,600
- Job Projections for 2028: 2,800 (increase of 8%)

Geometry Concepts

- Apply volume of solids.
- Apply concepts of density.
- Apply geometric methods to solve design problems.

Is this a good career for me?

Agricultural engineers:

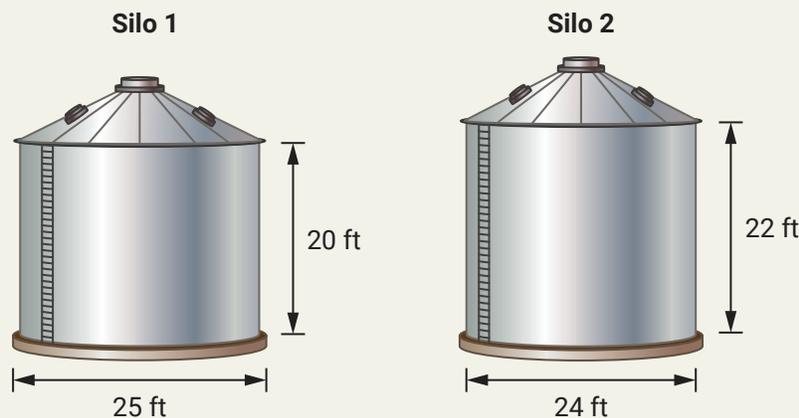
- Use computers to design equipment, systems, or structures
- Modify factors that affect production
- Test equipment
- Oversee construction and production operations
- Document technical design details.

Lesson Objective

In this lesson, you will look at how an agricultural engineer uses the volume of solids when designing and evaluating structures and systems used in agricultural settings.

1 Step Into the Career: Volume of Cylinders

An agricultural engineer is designing a farm storage system that will contain a silo for storing dried, shelled corn. The cylindrical part of the silo should store up to 400,000 pounds of corn. If the corn weighs 42 pounds per cubic foot, then which silo should be used?



Devise a Plan

Step 1: Find the storage capacity of each silo. The storage capacity is the volume of the cylindrical part of the silo. The formula for the volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.

Step 2: Find the weight of the corn that can be stored in each silo.

Step 3: Determine which silo can store 400,000 pounds of corn.

Walk Through the Solution

Step 1: Find the volume of the cylindrical part of each silo.

Since the diameters of the silos are given, use $\frac{25 \text{ ft}}{2} = 12.5 \text{ ft}$ for the radius of Silo 1 and $\frac{24 \text{ ft}}{2} = 12 \text{ ft}$ for the radius of Silo 2.

Capacity of Silo 1

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(12.5^2)(20) \\ &= 3125\pi \\ &\approx 9817 \text{ ft}^3 \end{aligned}$$

Capacity of Silo 2

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(12^2)(22) \\ &= 3168\pi \\ &\approx 9953 \text{ ft}^3 \end{aligned}$$

Step 2: Find the weight of the corn that can be stored in each silo. Multiply each volume by the weight per unit of volume, which is 42 pounds per cubic foot.

Silo 1: $(9817 \text{ ft}^3)(42 \text{ lb/ft}^3) = 412,314 \text{ lb}$

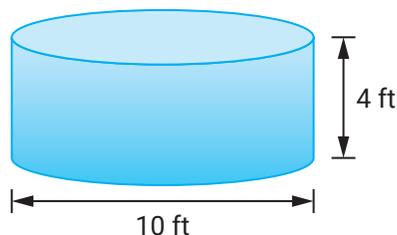
Silo 2: $(9953 \text{ ft}^3)(42 \text{ lb/ft}^3) = 418,026 \text{ lb}$

Step 3: Determine which silo can store 400,000 pounds of corn.

Either silo can store 400,000 pounds of corn.

On the Job: Apply Volume of Cylinders

1. An agricultural engineer is designing a commercial fishery that will raise tilapia. The fishery will have cylindrical aquaculture tanks with the dimensions shown.



Each tilapia requires
3 gallons of water.

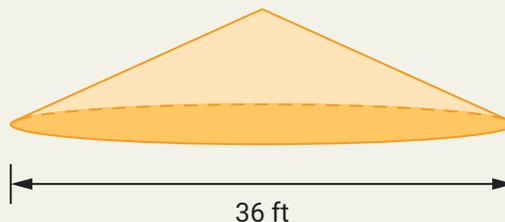
- What is the volume of a tank? Round to the nearest cubic foot.
- If 1 cubic foot of water is about 7.5 gallons, then what is the capacity of the tank in gallons?
- If each tilapia requires 3 gallons of water, then how many tilapia can be placed in a tank?

2 Step Into the Career: Volume of Cones

An agricultural engineer designs an area for temporary storage of 2800 cubic feet of harvested wheat kernels. A cone-shaped pile of 2800 cubic feet of wheat kernels will have a diameter of 36 feet. At what minimum height above the ground should the end of the grain auger transporting the wheat be set so that it clears the pile?



The volume of the pile of wheat kernels is 2800 ft³.



Devise a Plan

The end of the auger should be set at a height equal to or greater than the height of the pile of wheat kernels. To find the minimum height for the end of the auger, find the height of the pile of wheat kernels.

Step 1: Write the formula for the volume of a cone. The formula for the volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.

Step 2: Substitute known values into the formula.

Step 3: Solve for the height.

Walk Through the Solution

The volume of the cone-shaped pile of wheat kernels is 2800 cubic feet. Since the diameter of the pile is 36 feet, the radius is $\frac{36 \text{ ft}}{2} = 18 \text{ ft}$.

$$V = \frac{1}{3}\pi r^2 h$$

Formula for the volume of a cone

$$2800 = \frac{1}{3}\pi(18^2)h$$

Substitute 2800 for V and 18 for r .

$$2800 = 108\pi h$$

Simplify.

$$\frac{2800}{108\pi} = h$$

Divide each side by 108π .

$$8.25 \approx h$$

Simplify.

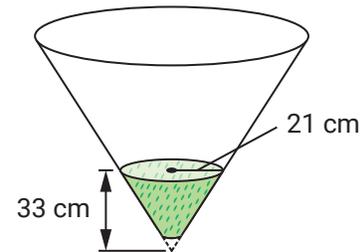
The height of the pile of wheat kernels is about 8.25 feet. So, the end of the grain auger should be set at a minimum height of 8.25 feet above the ground.

On the Job: Apply Volume of Cones

2. A cone-shaped seed spreader has a gate that controls the rate at which seeds flow out of the spreader. An agricultural engineer conducts experiments to determine which gate setting will have a flow rate around 3500 cubic centimeters per second for rye grass seeds.

The engineer records the times it takes for the same amount of rye grass seeds, which are shown in the table, to empty out of the container at different gate settings.

Gate Setting	Time to Empty (s)	Flow Rate (cm ³ /s)
1.5	11.8	?
2.0	4.1	?
2.5	2.0	?



- What is the volume of the seeds in the spreader? Estimate the volume using the volume of a cone with the dimensions shown. Round to the nearest cubic centimeter.
- Find the flow rate for each gate setting to the nearest cubic centimeter per second.
- Which gate setting should be used?

3 Step Into the Career: Volume of Spheres

An irrigation system is being designed to contain underground water storage tanks like the one shown. The manufacturer says that the tank needs to be at least 25% full at all times to prevent any movement or distortion of the tank.

If the diameter of the spherical storage space of the tank is 70 inches, then what is the minimum number of gallons of water that need to be in the tank at all times? (Use $231 \text{ in}^3 = 1 \text{ gal.}$)



Devise a Plan

Step 1: Find the volume of the spherical storage space. The formula for the volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

Step 2: Convert the volume to gallons.

Step 3: Find 25% of the total volume in gallons.

Walk Through the Solution

Step 1: Find the volume of the spherical storage space. Since the diameter is 70 inches, the radius is $\frac{70 \text{ in.}}{2} = 35 \text{ in.}$

$$V = \frac{4}{3}\pi r^3 \quad \text{Formula for the volume of a sphere}$$

$$= \frac{4}{3}\pi(35)^3 \quad \text{Substitute 35 for } r.$$

$$\approx 179,594 \quad \text{Simplify.}$$

The volume of the spherical storage space is about 179,594 cubic inches.

Step 2: Convert the volume of the storage space in cubic inches to gallons.
Use $231 \text{ in}^3 = 1 \text{ gal.}$

$$179,594 \text{ in}^3 \cdot \frac{1 \text{ gal}}{231 \text{ in}^3} \approx 777.5 \text{ gal}$$

Step 3: Find 25% of the capacity of the tank.

$$777.5 \cdot 0.25 \approx 194.4 \text{ gal}$$

There should be at least 194.4 gallons in the tank at all times.

On the Job: Apply Volume of Spheres

3. Water is often used to transport harvested tomatoes through a cleaning and packaging process. This process is most efficient when the tomatoes float. So, the density of the tomatoes should be less than the density of the water, which is 1 gram per cubic centimeter.

An agricultural engineer collects data for a sample of cherry tomatoes. The table shows the data for the tomatoes with the least and greatest masses.

Mass (g)	Diameter (cm)	Density (g/cm ³)
9	2.7	?
15	3.3	?



- What is the volume of each tomato? Assume that the tomatoes are spheres. Round to the nearest tenth.
- Find the density of each tomato by dividing its mass by its volume.
- Compare the density of each tomato to the density of water. Will each tomato float in water?

Career Spotlight: Practice

4. To help with irrigating farmland, an agricultural engineer is planning the transportation of water using tankers like the one shown.



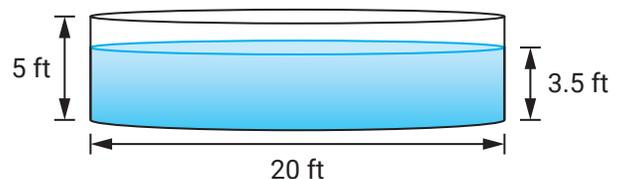
- The cylindrical tank shown has a radius of 3 feet and a height of 38 feet. What is the volume of the tank to the nearest gallon? (Use $1 \text{ ft}^3 \approx 7.5 \text{ gal.}$)
 - Due to weight limits, the truck can carry a maximum of 70,000 pounds of water. If the density of the water being transported is 8.3 pounds per gallon, does the truck with a full tank exceed the weight limit? Explain.
5. Onions that are more than 90% water need special handling since they are more susceptible to bruising. The table shows data collected for two onions. Find the percent of water in each onion. Do either of the onions need special handling? Assume that the onions are spheres, and use $1 \text{ cm}^3 = 1 \text{ mL}$.

QUICK TIP

The percent of an onion that is water can be found as follows: $\frac{\text{water content (mL)}}{\text{volume of onion (mL)}} \cdot 100\%$.

Onion	Diameter (cm)	Water Content (mL)	Percent of Onion That Is Water
Yellow onion	7	160	?
Sweet onion	6	105	?

6. In an irrigation system for a nursery, water is pumped from a well at a rate of 80 gallons per minute and held in a cylindrical tank. Suppose the height of the water in the tank is 3.5 feet. How long will it take to fill the tank to the top? (Use $1 \text{ ft}^3 \approx 7.5 \text{ gal.}$)



Devise a Plan

Step 1: Find the volume of the portion of the cylindrical tank that does not contain water.

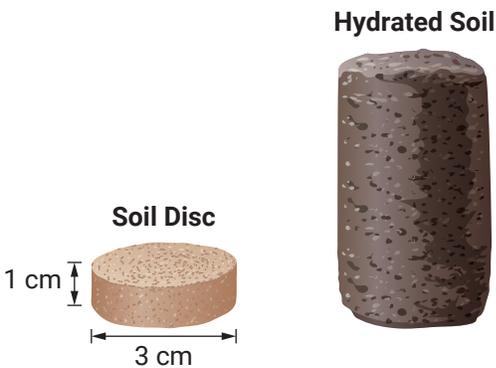
Step 2: _____?

Step 3: _____?

7. A bumper crop of soybeans has been stored in two cone-shaped piles. One pile has a diameter of 36 feet and a height of 8.4 feet. The other pile has a diameter of 24 feet and a height of 5.6 feet. If the soybeans weigh 47 pounds per cubic foot, then how many pounds of soybeans are stored in both piles?

Career Spotlight: Check

8. A nursery uses dehydrated soil discs to start seeds. Each disc is watered to expand to a cylinder of soil as shown. A hydrated cylinder of soil has 7 times the volume of a disc. Then a seed is planted in the hydrated soil.



Select the answer from each box that makes the sentence true.

An agricultural engineer wants to know how much water is needed to hydrate a soil disc.

First, she finds the volume of the soil disc, which is about

- | |
|---------|
| a. 4.7 |
| b. 7.1 |
| c. 28.3 |

cubic centimeters.

The volume of the hydrated soil is 7 times the volume of the disc, so the volume of the

water is

- | |
|------|
| a. 6 |
| b. 7 |
| c. 8 |

times the volume of the disc. Since $1 \text{ cm}^3 = 1 \text{ mL}$, the amount of water

in the hydrated soil is about

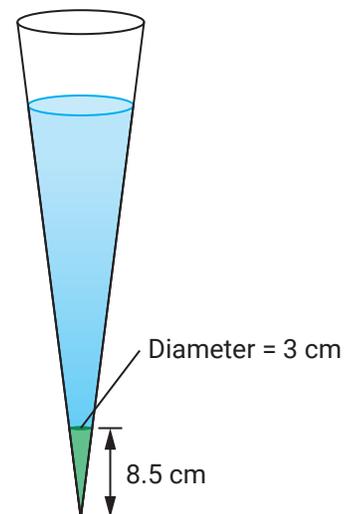
- | |
|---------|
| a. 28.2 |
| b. 32.9 |
| c. 42.6 |

milliliters.

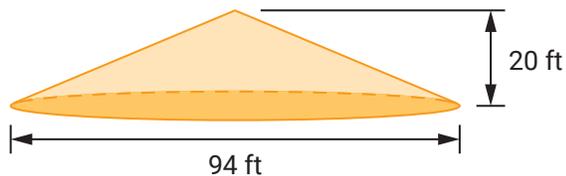
9. A sample of 1000 milliliters of water is taken from a water source that will be used for an irrigation system. The water is put in a sedimentation cone, and the contents settle for 30 minutes.

The diagram shows the cone-shaped pile of sediment that has settled at the bottom of the cone. What percent of the water is sediment? (Use $1 \text{ cm}^3 = 1 \text{ mL}$.)

- A. about 2%
- B. about 5%
- C. about 20%
- D. about 98%



10. Two grain silos each have cylinder-shaped storage spaces that have the same volume. The storage space for one silo has a diameter of 30 feet and a height of 24 feet. The storage space for the other silo has a diameter of 26 feet. What is the height of this storage space to the nearest foot?
11. The cone-shaped pile of corn shown will be transported using a grain truck that can hold 880 cubic feet. The corn weighs 42 pounds per cubic foot.



The grain truck holds 880 ft³.



Select all the statements that are true.

- The radius of the pile of corn is 94 feet.
 - The volume of the pile of corn rounded to the nearest hundred is 46,300 cubic feet.
 - The pile of corn weighs less than 2,000,000 pounds.
 - It will take 46 truckloads to transport all of the corn.
 - The possible dimensions of the grain truck's storage space is 4 feet by 25 feet by 8 feet.
12. A dairy is processing milk to make mozzarella cheese balls. Each cheese ball needs to have between 52% and 60% water content to meet regulations. The data for three samples are shown. Find the percent of each sample that is water. Assume that the samples are spheres, and use $1 \text{ cm}^3 = 1 \text{ mL}$. Match each sample to the correct percent of water content.

Sample	Diameter (cm)	Water Content (mL)
Sample 1	5.6	52
Sample 2	5.2	37
Sample 3	6.0	60

	50%	53%	57%
Sample 1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Sample 2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Sample 3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Notes

TEACHER'S EDITION

Pathway2 Geometry



PATHWAY 2 GEOMETRY
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Preview Sample



NS4ed

TEACHER'S EDITION



PATHWAY 2 GEOMETRY
Education with Destination

Pathway2 Geometry





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PATHWAY 2 GEOMETRY
Education with Destination

Pathway2Geometry

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			CCSS	Occupation	
CHAPTER 6 Polygons and Quadrilaterals	6.1	Properties and Conditions for Parallelograms	149	G-CO.11	Mechanical Drafter
	6.2	Conditions for Special Quadrilaterals	157	G-CO.11	Motorcycle Mechanic
	6.3	Identifying Quadrilaterals on the Coordinate Plane	165	G-CO.11	Fashion Designer
	6.4	Perimeter on the Coordinate Plane	177	G-GPE.7, G-MG.3	Fence Erector

			CCSS	Occupation	
CHAPTER 7 Proportions and Similarity	7.1	Apply Dilations	187	G-SRT.1a, G-SRT.1b, G-MG.3	Advertising and Promotions Manager
	7.2	Properties of Similar Figures	193	G-SRT.2, G-MG.3	Set and Exhibit Designer
	7.3	Use Similar Triangles	201	G-SRT.5	Forester
	7.4	Use Proportionality Theorems	209	G-SRT.5	Urban and Regional Planner

			CCSS	Occupation
CHAPTER 8 Right Triangles and Trigonometry	8.1	Apply the Pythagorean Theorem	219	G-SRT.8 Construction and Building Inspector
	8.2	Use Similar Right Triangles	227	G-SRT.5 Photographer
	8.3	Trigonometry with Right Triangles	235	G-SRT.7, G-SRT.8 Solar Photovoltaic Installer
	8.4	Trigonometry with All Triangles	243	G-SRT.10, G-SRT.11 Sound Engineering Technician

			CCSS	Occupation
CHAPTER 9 Circles	9.1	Central Angles and Inscribed Angles	251	G-C.2 Security and Fire Alarm Systems Installer
	9.2	Segment Relationships in Circles	259	G-C.2 Life, Physical, and Social Science Technician
	9.3	Circumference, Arc Length, and Radian Measure	267	G-GMD.1, G-C.5 Mechanical Engineering Technician
	9.4	Equations of Circles	273	G-GPE.1 Geoscientist

			CCSS	Occupation
CHAPTER 10 Areas	10.1	Area of Triangles	281	G-SRT.9 Surveyor
	10.2	Area of Polygons	287	G-MG.2 Fish and Game Warden
	10.3	Areas of Circles and Sectors	293	G-GMD.1, G-C.5 Cardiovascular Technologist and Technician
	10.4	Area of Composite Figures	299	G.MG.1, G.MG.2 Appraiser and Assessor of Real Estate
	10.5	Area in the Coordinate Plane	307	G-GPE.7 Computer Specialist

			CCSS	Occupation	
CHAPTER 11 Surface Area and Volume	11.1	Visualizing Solids	317	G-GMD.4	Architectural and Civil Drafter
	11.2	Surface Area of Prisms and Pyramids	327	G-MG.1	Anthropologist and Archeologist
	11.3	Surface Area of Cones, Cylinders, and Spheres	333	G-MG.3	Industrial Production Manager
	11.4	Volume of Prisms and Pyramids	343	G-GMD.3	Heating, Air Conditioning, and Refrigeration Mechanic and Installer
	11.5	Volume of Cylinders, Cones, and Spheres	349	G-GMD.3, G-MG.2	Agricultural Engineer

			CCSS	Occupation	
CHAPTER 12 Probability and Decision Making	12.1	Disjoint and Overlapping Events	357	S-CP.4, S-CP.7	Health Educator
	12.2	Conditional Probability	365	S-CP. 3, S-CP. 5	Personal Financial Advisor
	12.3	Independent Events	371	S-CP.2, S-CP.4	Information Security Analyst
	12.4	Dependent Events	379	S-CP.8	Gaming Manager

LESSON 1.3

Use Theorems About Angles



Common Core State Standards

G-CO.9 Prove theorems about lines and angles.

Mathematical Practices 1, 2, 4

CAREER SPOTLIGHT: Carpenter

Carpenters use science and math with a focus on constructing, repairing, and installing building frameworks and structures made from wood and other materials. Carpenters use measurements and angles to make sure that components of a structure fit together correctly to ensure a sturdy structure.

- Discuss carpentry with students by reading the Career Spotlight together.
- Find local colleges and vocational technical schools with carpentry programs to share with students.
- Research local companies that employ carpenters and ask what they do for the companies.

Video: Carpenter

Have students watch this video, which describes the types of projects a carpenter might work on.

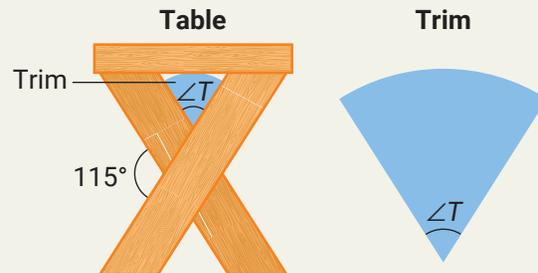
Lesson Objective

In this lesson, you will look at how a carpenter applies theorems about angles to carpentry tasks such as framing buildings and building furniture.

Teaching Support

1 Step Into the Career: Linear Pair Theorem

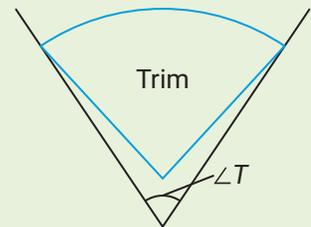
A carpenter is building coffee tables with crossed legs and decorative trim. One version is shown. What is the angle measure that the carpenter should cut the bottom angle of the trim, $\angle T$, so that it fits with no gaps?



Guiding Questions

- In Step 3, why can the Linear Pair Theorem be applied to this situation?
- In Step 4, can the trim be cut with other angle measures so that it fits with no gaps?

EXTENSION Suppose the trim was not cut to the correct angle. As a result, there is a V-shaped gap at the bottom, as shown. Is the angle used for the trim smaller or larger than the angle that fit with no gaps? Explain. (Answer: larger; Possible explanation: If you draw a line across the top, you have two isosceles triangles. The base angles of the incorrect trim are smaller than what the base angles of the trim should be, so the vertex angle of the incorrect trim is larger.)



On the Job: Apply The Linear Pair Theorem

Answers

1a. no

1b. 54°

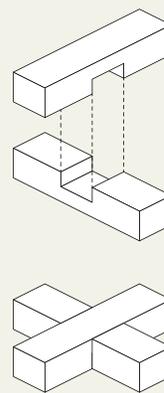
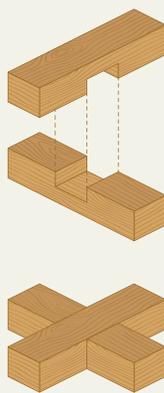
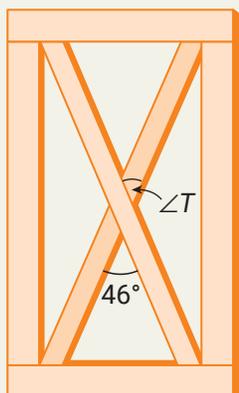
Use these questions to check students' understanding.

- In 1a, how did you check whether the 57° trim would fit with no gaps?
- In 1b, how did you determine what angle the trim should be cut to?

2 Step Into the Career: The Vertical Angles Theorem

A carpenter is building a shelving unit with additional support on each side provided by two boards that are connected with edge cross lap joints, as shown.

What is the measure of $\angle T$ that the carpenter should cut the notch in the board?



In edge cross lap joints, matching notches are cut in the two boards so that they fit together snugly.

As students may not be familiar with edge cross lap joints, direct them to the diagram showing how the matching notches are joined.

Guiding Questions

- In Step 3, why can the Vertical Angles Theorem be applied in this situation?
- In Step 4, could there be other angles that would work for the notches in the two boards to fit together?

DIFFERENTIATION: ENRICHMENT Edge cross lap joints are useful in some situations, such as where you want two intersecting boards to be flush to another surface, but both boards are weakened by being cut halfway through. Ask students when edge cross lap joints might be a better option and when they might not be a good option. (Possible answer: Cross lap joints might be sturdier without needing additional reinforcement but might require greater precision to make and cost more. They might be better for smaller jobs such as cabinets and bookcases.)

On the Job: Apply The Vertical Angles Theorem

Answers

2a. no

2b. 58°

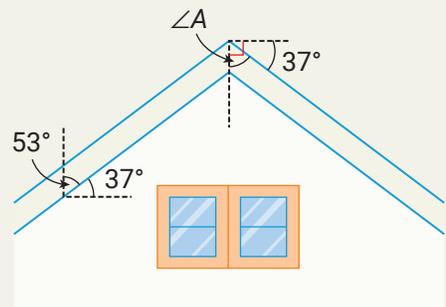
2c. The apprentice used the measure of the complement of the given angle.

Use these questions to check students' understanding.

- In 2a, how did you check whether the 32° would work for the box?
- In 2b, how did you determine the correct angle for the notch?
- In 2c, what is the relationship between a 58° angle and a 32° angle?

3 Step Into the Career: Congruent Complements Theorem

A carpenter is framing the roof of a house where the pitch of the roof is 37° , as shown. What angle measure should be used for $\angle A$?



Students may not be familiar with the pitch of a roof, which is a measure of the steepness of the roof. The pitch of a roof can be given as an angle from the horizontal, as done here, or as a ratio of the vertical rise of the roof to the horizontal distance spanned. Here, an angle of 37° corresponds to a ratio that is about $9 : 12$, which means that the roof would rise 9 feet for every 12 feet of horizontal distance.

Guiding Questions

- In Step 1, how can a relationship between the 37° angle and the 53° angle be determined?
- In Step 2, what information shows the relationship between the 37° angle and $\angle A$?

DIFFERENTIATION: ENRICHMENT Different pitches of roofs are used in different circumstances depending on many factors, including the type of building (commercial buildings often have roofs that are less steep) and the environment (roofs that colder places are often steeper so that snow does not build up). Ask students to think of other factors that could affect the desired pitch of a roof. (Possible answer: other environmental factors such as rain, wind, and surrounding foliage, type of roofing materials, cost of materials)

On the Job: Apply The Congruent Supplements Theorem

Answers

- 3a. They are supplementary angles.
3b. They are supplementary angles.
3c. 54°

Use these questions to check students' understanding.

- In 3a, how did you determine the relationship between the angles? Is there more than one way to do this?
- In 3b, how did you determine the relationship between the angles? Is there more than one way to do this?

Career Spotlight: Practice

Solution Steps for Exercises 4–6

These steps will help guide students in solving these practice exercises.

Exercise 4

Answer

4. 12°

Solution Steps

- Determine that $\angle J$ and the 168° angle form a linear pair.
- Write an equation based on the relationship between the 168° angle and $\angle J$.
($168^\circ + m\angle J = 180^\circ$)
- Solve the equation to find $m\angle J$. (12°)

Exercise 5

Answer

5. 120°

Devise a Plan

Possible plan:

Step 1: Determine the relationship between the 120° angle and $\angle R$.

Step 2: Use the relationship between the 120° angle and $\angle R$ to state a relationship between the angle measures.

Step 3: Find $m\angle R$.

Solution Steps

- The 120° angle and $\angle R$ are vertical angles.
- Vertical angles are congruent.
- Congruent angles have the same measure.

Exercise 6

Answers

- 6a. The angles are supplementary angles.
- 6b. The angles are supplementary angles.
- 6c. Both the 139° angle and $\angle T$ are supplementary angles to a 41° angle. By the Congruent Supplements Theorem, they are congruent.
- 6d. 139°

Solution Steps

- The angle labeled 139° and the angle labeled 41° on the left are supplementary angles.
- The angle labeled 41° and $\angle T$ on the right are supplementary angles.
- Angles that are supplementary to the same angle are congruent.
- $\angle T$ is congruent to the 139° angle.

Career Spotlight: Check

Tips for Completing Exercises 7–11

These tips will help students in solving these exercises and similar assessment items.

Exercise 7

Answer

7. b. complementary angles, b. complementary angles, b. 64°

Tip Encourage students to check their answers for reasonableness by reading the entire problem after they have chosen their answers. For example, since the sum of the measures of complementary angles is 90° , the number selected for the third blank cannot be greater than 90° .

Exercise 8

Answer

8. D

Tip Encourage students to examine the answer choices after reading the problem to eliminate choices that can easily be identified as incorrect. For example, the measures of both complementary angles must be less than 90° , so choice B can be eliminated.

Exercise 9

Answer

9. c, e

Tip Encourage students to determine and find the information that is needed to select the true statements. For example, students should determine the relationship between the angle labeled 78° and $\angle D$ in order to select the correct value for $m\angle D$.

Exercise 10

Answer

10. a, b, g

Tip Encourage students to determine and find the information that is needed to select the true statements. For example, students should determine the relationship between the angle labeled 68° and $\angle U$ in order to select the correct value for $m\angle U$.

Exercise 11

Answer

11. $\angle 1$: 34° , $\angle 2$: 112° , $\angle 3$: 112° , $\angle 4$: 68°

Tip Remind students that every angle measure will not necessarily match with an angle and an angle measure could match with more than one angle. Encourage students to list the relationships between the angles before determining the angle measures.

Notes

LESSON 3.2

Apply Reflections



Common Core State Standards

G-CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Mathematical Practices 1, 2, 4

CAREER SPOTLIGHT: Marine Engineer and Naval Architect

Marine engineers and naval architects design, build, and maintain ships, from sailboats to aircraft carriers. This work requires a degree in marine engineering or naval architecture as well as knowledge of arithmetic, algebra, geometry, calculus, and statistics. Marine engineers prepare detailed drawings and schematics and analyze data to determine the feasibility of product proposals. Naval architects design complete hulls and superstructures as well as the layouts of ship interiors, including cargo space, passenger compartments, ladder wells, and elevators.

- Discuss marine engineering and naval architecture with students by reading the Career Spotlight together.
- Find local colleges, universities, or state maritime academies with marine engineering or naval architecture programs to share with students.
- Research local companies that employ marine engineers and naval architects, and ask what they do for the companies.

Video: Marine Engineers and Naval Architects

Have students watch this video, which describes the types of projects a marine engineer or naval architect might work on.

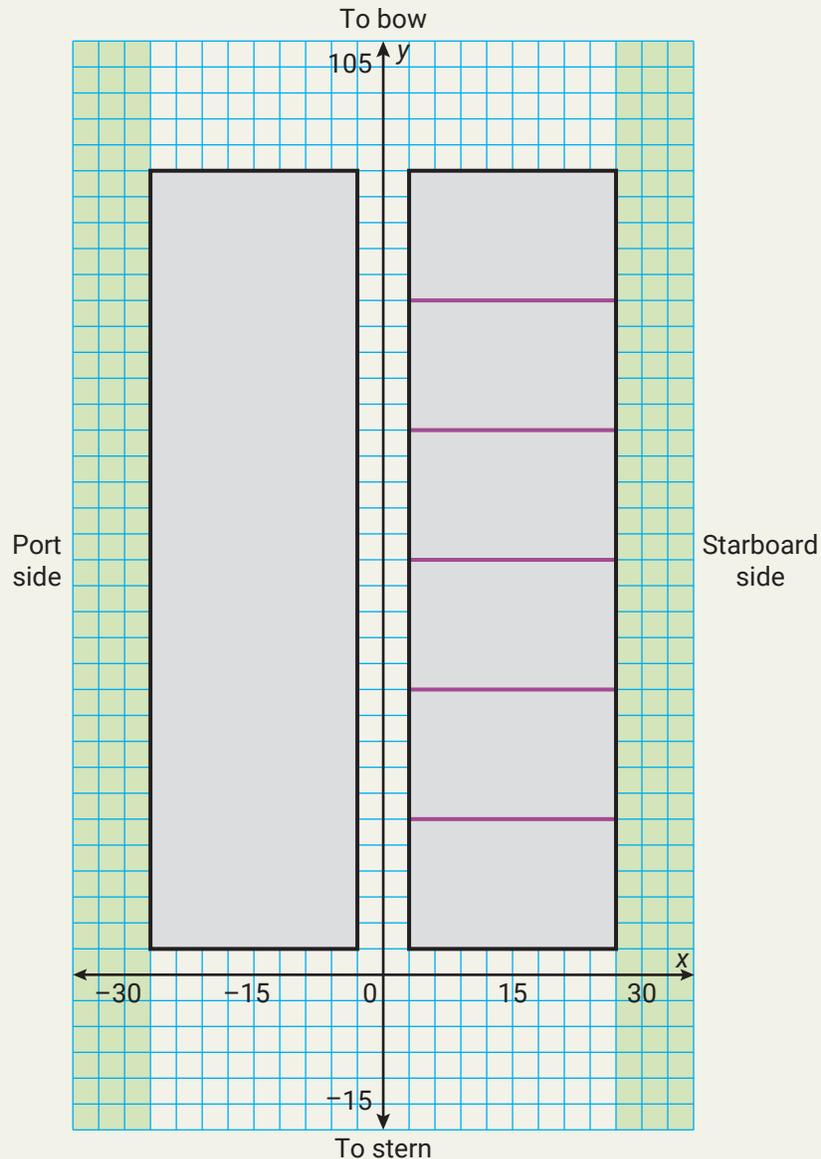
Lesson Objective

In this lesson, you will look at how a marine engineer or naval architect applies geometric reflections on a coordinate plane.

Teaching Support

1 Step Into the Career: Reflection Functions

Danica is a naval architect. She is sketching a plan for the hinges of the hatch covers of a cargo ship when the hatch covers are in the closed position. The hinges on the starboard side of the ship are shown. The hinges on the port side of the ship are a reflection across the y -axis. Draw the hatch cover hinges for the port side of the ship when the hatch covers are in the closed position.



Guiding Questions

- In Step 2, what are the coordinates of the endpoints of the starboard hatch cover hinge closest to the stern of the ship?
- In Step 3, how can you check that you have drawn the port hatch cover hinges correctly?

ENGLISH LANGUAGE LEARNER Rather than *front*, *back*, *left*, and *right*, which vary based on the position of an observer, nautical terms refer to the boat itself. The front of the boat, as it is intended to move through water, is the *bow*. The back of the boat is the *stern*. The left side is *port*, and the right side is *starboard*. In most boats, a person can face the bow while the boat is moving, keeping port on the left and starboard on the right. In boats like row boats, however, the bow is still the front of the boat as it is intended to move through water, but the rower is facing the stern, putting port on the rower's right and starboard on the rower's left.

On the Job: Apply Reflection Functions

Answer

1. reflection D

Use these questions to check students' understanding.

- Which coordinate is affected by the reflection across the x -axis?
- What are the coordinates of one divider on the port side?
- How do those coordinates change after the reflection?

2 Step Into the Career: Reflections on the Coordinate Plane

A marine engineer is working on plans to attach chainplates to a sailboat. His plans are placed on a coordinate plane. The coordinates of the starboard chainplates are $(1, -5)$, $(2, -4)$, and $(3, -3)$. The coordinates of the port chainplates can be found using the reflection $R(x, y) = (y, x)$. What are the coordinates of the port chainplates? What is the line of reflection?



Guiding Questions

- In Step 2, how can you use a compass and straightedge to construct a perpendicular bisector?
- In Step 3, how can you find the equation of the line of reflection?

On the Job: Apply Reflections on the Coordinate Plane

Answer

2. $(-80, 600), (-110, 0)$

Use these questions to check students' understanding.

- Which coordinate is affected by the reflection across the y -axis?
- How does the x -coordinate change in a reflection across the y -axis?

EXTENSION For the boat in this illustration, have students explain why they think the mast is or is not the line of reflection for the backstay and forestay. (Possible answer: The mast is not a line of reflection because it appears that the end of the backstay is farther away from the mast than the end of the forestay.)

Career Spotlight: Practice

Solution Steps for Exercises 3–5

These steps will help guide students in solving these practice exercises.

Exercise 3

Answer

3. $y = -0.84x + 1.2$

Solution Steps

- Determine which coordinate is affected by the reflection across the x -axis. (y -coordinate)
- Determine two points on the line with equation $y = 0.84x - 1.2$. (Possible answer: $(0, -1.2)$, $(2, 0.48)$)
- Determine two points on the reflected line. (Possible answer: $(0, 1.2)$, $(2, -0.48)$)
- Find the equation. ($y = -0.84x + 1.2$)

Exercise 4

Answers

4a. (25, 0), (0, 250), and (-50, 300)

4b. $x = 75$

Devise a Plan

Possible plan:

Step 1: Determine the coordinates of the other corners.

Step 2: Plot both sets of coordinates, connect each point to its transformation, and construct the perpendicular bisector of each segment.

Step 3: Identify the perpendicular bisector.

Solution Steps

- Determine which coordinate is affected by the transformation $R(x, y) = (150 - x, y)$. (x-coordinate)
- Plot both sets of coordinates, connect each point to its transformation, and construct the perpendicular bisector of each segment, or calculate the midpoint of each segment that connects a point to its transformation. ((75, 0), (75, 250), and (75, 300))
- Determine whether all of the perpendicular bisectors are the same line. (They are the same line.)
- Identify the perpendicular bisector. ($x = 75$)

Exercise 5

Answer

5. (-30, 45), (-20, 30)

Solution Steps

- Determine a function for the reflection. ($R_{y=-x}(x, y) = (-y, -x)$)
- Apply the function. ((-30, 45), (-20, 30))

Career Spotlight: Check

Tips for Completing Exercises 6–9

These tips will help students in solving these exercises and similar assessment items.

Exercise 6

Answer

6. D

Tip Encourage students to draw the reflections on a coordinate plane and then match the correct choice to their drawing.

Exercise 7

Answer

7. Piece 1: no quadrant; Piece 2: quadrant IV; Piece 3: quadrant I; Piece 4: quadrant III

Tip Encourage students to perform the reflections on a particular piece of the quadrant, such as the stairs, instead of trying to perform the reflections on the entire drawing of the elevators in quadrant II.

Exercise 8

Answer

8. D

Tip Encourage students to use the given information shown in the diagram. Students should find the perpendicular bisectors of the two segments that connect corresponding points. The line of reflection is the line between Line 1 and Line 3.

Exercise 9

Answer

9. a, b, d, e, f

Tip Encourage students to plot the corners and lines of reflection on a coordinate grid. Students should check their answers by using the functions.

LESSON 11.5

Volume of Cylinders, Cones, and Spheres



Common Core State Standards

G-GMD.3 Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.

G-MG.2 Apply concepts of density based on area and volume in modeling situations.

Mathematical Practices 1, 2, 4

CAREER SPOTLIGHT: Agricultural Engineer

Agricultural engineers use science and math with a focus on designing agricultural systems related to farming practices. This career draws from principles in other engineering fields, such as civil and mechanical engineering, and combines them with environmental sciences and biology to improve the production of plants and livestock.

- Discuss agricultural engineering with students by reading the Career Spotlight together.
- Find local colleges and universities with an agricultural engineering program to share with students.
- Research local companies that employ agricultural engineers, and ask what they do for the companies.

Video: Agricultural Engineers

Have students watch this video, which describes the types of projects agricultural engineers might work on.

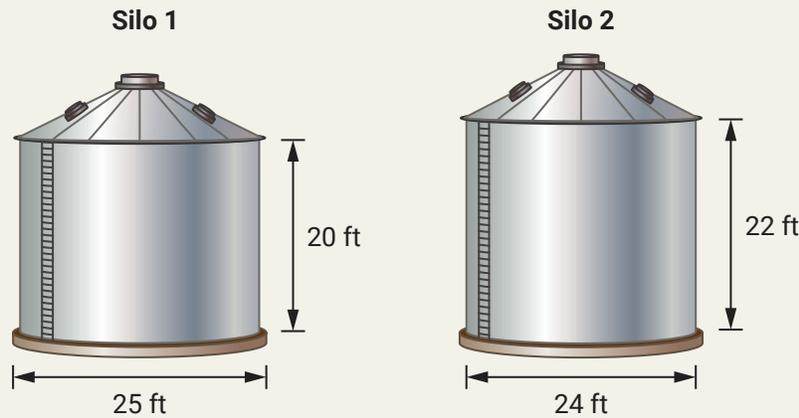
Lesson Objective

In this lesson, you will look at how an agricultural engineer uses the volume of solids when designing and evaluating structures and systems used in agricultural settings.

Teaching Support

1 Step Into the Career: Volume of Cylinders

An agricultural engineer is designing a farm storage system that will contain a silo for storing dried, shelled corn. The cylindrical part of the silo should store up to 400,000 pounds of corn. If the corn weighs 42 pounds per cubic foot, then which silo should be used?



Guiding Questions

- In Step 2, how can the number of pounds be found using a proportion?
- In Step 3, will a silo that has a diameter of 26 feet and a height of 20 feet hold the corn?

EXTENSION In this example, both silos can store 400,000 pounds of corn. Ask students to suggest what other criteria an agricultural engineer could consider in designing a silo. Discuss what considerations could be made about the amount of material needed to build the silos. Remind students that the lateral surface area of a cylinder can be determined by the formula $S = 2\pi rh$, where r is the radius and h is the height. Ask students to determine which silo has the greater lateral surface area. (Answer: Silo 2)

TECHNOLOGY Challenge students to think about the dimensions of a cylinder that can hold 400,000 pounds of corn with the least amount of lateral surface area. Have students find the volume needed for 400,000 pounds of corn, and then ask for an expression for the height h of a cylinder that can hold the corn in terms of radius r . Demonstrate using technology (by graphing or by using a spreadsheet) how to determine the radius that results in the minimum lateral surface area.

On the Job: Apply Volume of Cylinders

Answers

1a. 314 ft^3

1b. about 2356 gallons

1c. 785 tilapia

Use these questions to check students' understanding.

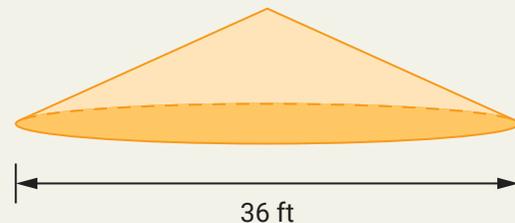
- In 1a, what formula did you use to find the volume? How did you determine the radius?
- In 1a, why is the volume of the tank in cubic feet an estimate?
- In 1b, how did you convert cubic feet to gallons?

2 Step Into the Career: Volume of Cones

An agricultural engineer designs an area for temporary storage of 2800 cubic feet of harvested wheat kernels. A cone-shaped pile of 2800 cubic feet of wheat kernels will have a diameter of 36 feet. At what minimum height above the ground should the end of the grain auger transporting the wheat be set so that it clears the pile?



The volume of the pile of wheat kernels is 2800 ft^3 .



Students may not be familiar with a grain auger. A grain auger is a tube with a spiral shaft in the middle that transports grain. This is not to be confused with an auger that is a type of drilling device.

Guiding Questions

- Could the height be determined if only the volume of the pile was given and not the diameter?
- The height and diameter of a pile of wheat must remain in proportion. Suppose the volume of the wheat increases. Will the height of the pile be greater than or less than 8.25 feet? Explain.

DIFFERENTIATION: ENRICHMENT The shape of a conical pile depends on the material. For wheat, the angle formed by a line from the vertex of the cone to the ground is about 25° . In this example, students can find this angle by calculating $\tan^{-1}\left(\frac{8.25}{18}\right) \approx 25^\circ$. If the grain for this pile is barley, the angle is about 28° . Ask students to determine the height and volume of a pile of barley with a diameter of 36 feet. (Answer: height: about 9.57 ft, volume: about 3247 ft^3)

On the Job: Apply Volume of Cones

Answers

2a. $15,240 \text{ cm}^3$

2b. for 1.5: $1292 \text{ cm}^3/\text{s}$, for 2.0: $3717 \text{ cm}^3/\text{s}$, for 2.5: $7620 \text{ cm}^3/\text{s}$

2c. setting 2.0

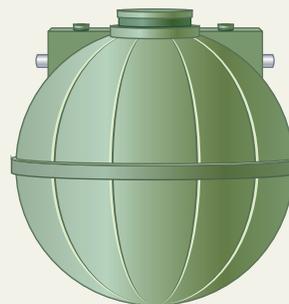
Use these questions to check students' understanding.

- In 2a, what volume formula did you use?
- In 2a, is the estimate for the volume of the seeds less than or greater than the actual amount of seeds? How do you know?
- In 2b, how did you find each flow rate?

3 Step Into the Career: Volume of Spheres

An irrigation system is being designed to contain underground water storage tanks like the one shown. The manufacturer says that the tank needs to be at least 25% full at all times to prevent any movement or distortion of the tank.

If the diameter of the spherical storage space of the tank is 70 inches, then what is the minimum number of gallons of water that need to be in the tank at all times? (Use $231 \text{ in}^3 = 1 \text{ gal.}$)



Guiding Questions

- In Step 2, how can the number of gallons be found using a proportion?
- In Step 3, suppose the tank needs to be at least 30% full. What is the minimum number of gallons that should be in the tank at all times?

On the Job: Apply Volume of Spheres

Answers

3a. 10.3 cm^3 , 18.8 cm^3

3b. about 0.87 g/cm^3 , about 0.80 g/cm^3

3c. yes

Use these questions to check students' understanding.

- In 3a, what volume formula did you use?
- In 3a, how did you find the radius?
- In 3b, how can the given units for the density help you find the densities?

Career Spotlight: Practice

Solution Steps for Exercises 4–7

These steps will help guide students in solving these practice exercises.

Exercise 4

Answers

4a. 8058 gal

4b. no; Possible explanation: 8058 gallons of water with a density of 8.3 pounds per gallon will weigh $8058 \cdot 8.3 = 66,881.4$ pounds. Since $66,881.4 < 70,000$, a full tank of water will not exceed the weight limit.

Solution Steps

- Use the volume formula for a cylinder to find the volume with $r = 3$ feet and $h = 38$ feet. (1074.4 ft^3)
- Multiply the volume by 7.5 to find the capacity in gallons. (about 8058 gal)
- Multiply the capacity in gallons by the density of water (8.3 lb/gal) to find the weight of water. (66,881.4 lb)
- Compare the weight of water to the weight limit. (The weight of water is under the weight limit.)

Exercise 5

Answer

5. yellow: 89%, sweet: 93%; The sweet onion needs special handling.

Solution Steps

- Use the volume formula of a sphere to find the volume of each onion. (180 cm^3 , 113 cm^3)
- Write the volume of each onion in milliliters. (180 mL, 113 mL)
- Find the percent of water in each onion. (89%, 93%)
- Determine whether each percent of water content is greater than 90%. (The sweet onion has a percent of water content greater than 90%.)

Exercise 6

Answer

6. about 44 minutes

Devise a Plan

Possible plan:

Step 1: Find the volume of the portion of the cylindrical tank that does not contain water.

Step 2: Convert the volume in cubic feet to gallons.

Step 3: Use the rate to find the time.

Solution Steps

- Use the volume formula for a cylinder to find the volume of the portion of the tank that does not contain water. (471.2 ft^3)
- Multiply the volume of the tank that is not water by 7.5 gal/ft^3 to convert to gallons. (3534 gal)
- Divide the number of gallons left to fill by 80 gal/min to find time to fill tank in minutes. (44 min)

Exercise 7

Answer

7. about 173,642 lb

Solution Steps

- Use the volume formula for a cone to find the volume of each pile. (2850.05 ft^3 , 844.46 ft^3)
- Find the sum of the volumes. (3694.51 ft^3)
- Multiply the sum by 47 lb/ft^3 to find the total weight of the piles. (173,642 lb)

Career Spotlight: Check

Tips for Completing Exercises 8–12

These tips will help students in solving these exercises and similar assessment items.

Exercise 8

Answer

8. b. 7.1, a. 6, c. 42.6

Tip Encourage students to check their answers for reasonableness by reading the entire problem after they have chosen their answers. For example, the product of the numbers selected for the first and second boxes should equal the number selected for the third box.

Exercise 9

Answer

9. A

Tip Encourage students to examine the answer choices after reading the problem to eliminate choices that can easily be identified as incorrect. For example, the diagram shows that the sediment does not appear to be 98% of the water, so choice D can be eliminated.

Exercise 10

Answer

10. 32 ft

Tip Encourage students to check their answer by calculating the volumes of both silos to make sure the volumes are equal.

Exercise 11

Answer

11. b, c

Tip Encourage students to determine and find the information that is needed to select the true statements. For example, students should find the radius, volume, and weight of the pile of corn and the number of truckloads needed to transport the corn in order to select the true statements.

Exercise 12

Answer

12. Sample 1: 57%, Sample 2: 50%, Sample 3: 53%

Tip Encourage students to work efficiently when answering a matching problem. For example, by process of elimination, students only need to find the percent of water content for two samples.

Notes



Pathway2 Geometry



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